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A mathematical modeling proposal for a Multiple Tasks Periodic Capacitated Arc Routing Problem

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ABSTRACT
The countless accidents and incidents occurred at dams at the last years, propelled the development of politics related with dams safety. One of the strategies is related to the plan for instrumentation and monitoring of dams. The monitoring demands from the technical team the reading of the auscultation data, in order to periodically monitor the dam. The monitoring plan of the dam can be modeled as a problem of mathematical program of the periodical capacitated arcs routing program (PCARP). The PCARP is considered as a generalization of the classic problem of routing in capacitated arcs (CARP) due to two characteristics: 1) Planning period larger than a time unity, as that vehicle make several travels and; 2) frequency of associated visits to the arcs to be serviced over the planning horizon. For the dam's monitoring problem studied in this work, the frequent visits, along the time horizon, it is not associated to the arc, but to the instrument with which is intended to collect the data. Shows a new problem of Multiple tasks Periodic Capacitated Arc Routing Problem and its elaboration as an exact mathematical model. The new main characteristics presented are: multiple tasks to be performed on each edge or edges; different frequencies to accomplish each of the tasks; heterogeneous fleet; and flexibility for more than one vehicle passing through the same edge at the same day. The mathematical model was implemented and examples were generated randomly for the proposed model's validation.
Keywords-Dams safety, Monitoring, Periodic routing

I. Introduction

The safety of dams constitute a cause of concern for the society, since there are potential risks for the people and goods installed at the cities and valleys downstream (low river), even if there are remote possibilities of ruptures on this works. The natural aging process of this works or the tendency, frequently checked, for the occupancy of the cities or valleys downstream of the dams, as well as the increased perception of the associated risk to this type of structures, lead to higher requirements for the safety of dams. This translates, mainly, by the establishment and fulfillment of norms for operation and inspection programs, observation, auscultation (monitoring) and maintenance of dams [1].

According to [2], in Brazil, some accidents occurred in the last years. Most of them involving dams with small dimensions, not generating large consequences to the downstream.However, the rupture at June's 17th 2004, at the gravity dam in Camará - Brazil, with 50m of maximum height, located on the proximities of Campina Grande - Brazil, in Paraíba, caused around 9 deaths and big damages to the city of Alagoa Grande - Brazil with 600 people homeless and, around, 200 houses destroyed, even the ones located a few kilometers from the downstream.

According to [3], although the Camará's Dam have been constructed in a very altered joint of the rock, located around 4 m of depth and which caused the rupture, haven't received proper cares at the project phase, or received any type of instrumentation. In September 2010, have been approved in Brazil the law nº 12334, at the safety area of dams, which demands that any dam's owner to install some control instruments and perform periodical field inspections for detection of eventual anomalies in its behavior.

The accidents and incidents that can happen on a dam are related to a large number of anomalies that can be noticed and, for this, demands the establishment of a system for prioritization and decentralization of the maintenance actions.

The objective of a safety evaluation is to determine the related conditions to the structural and operational safety of a dam. The evaluation must identify the problems and recommend the correctional repairs, operational restrictions and/or changes regarding the analysis and studies to determine solutions. In this sense, a continuous effort demands from the dam's owner inspections and periodical evaluations to ensure the safety during all the existence of the structure. The inspection is an extensive observation of the physical and visible elements of the dam and associated structures.

As can be verified in [4], the safety actions consists, basically, in routine and periodical inspections, reading and analysis of instrumentation, evaluation of potential risks, preventive and corrective maintenance of the hydromechanical equipments and training. The works must be inspected and observed lifelong, so it can allow the control of its safety conditions and operability.

In this sense the monitoring of the dam's instrumentation allows to detect, in good time, the development of an eventual deterioration scenario and detection of any anomaly, allowing to take appropriate corrective measures, to avoid the development of a bad scenario for the dam or, at least, lower the consequences. Therefore, so for the dam's safety to be effective, is necessary a well structured planning to, both the start of the works, as the lifelong of the dams, to avoid accidents.

Following the principle of an effective development to the planning of the instrumentation's monitoring, installed along the dam, this work seeks to contribute with a methodology for the tactical planning of monitoring of the instruments for the safety of the dams. This planning must considerate the restrictions imposed by the owners of the dams, especially regarding the periodical inspections, and meet the requirements established by the safety rules. Such observations will lead to the development of a model of binary integer linear mathematical to solve the Periodic Capacitated Arc Routing Problem, associated to the dam's safety problem.

II. Periodical Capacitated Arcs Routing Problem - (PCARP)

The PCARP was defined by [5] considering a network $G = (N, A)$ and a discreet planning period H of s periods or days, on which any task (required arc) u has a number of services $f(u)$. This means that the task must be $f(u)$ times, $1 \leq f(u) \leq s$, but, at most, once a period or day, treated during the planning period H. The total number of services z to be accomplished in H is, next, the sum of all the task services. The PCARP consists in finding a minimum set of routes, which starts and ends at the depot, satisfying the necessary number of services in all the arcs without exceeding the vehicle's capacity.

In the PCARP, as defined by [5], all the arcs have a constant frequency requirement in all the planning period: or the task must be treated of $f(u)$ times on each one of the periods of s or days. Nonetheless, there are problems which are not regular, the frequency can be relative to an arc bigger than the period or one day and have arcs with frequencies in periods or days different from each other.

A proposal of a mathematical model for a PCARP is given by [6]. The authors presents a model considering a minimum and maximum spacing between two successive treatments for a task.

According to [7] the routing problems in periodical arcs are more poor and disorganized than the routing problems in which, in turn, are more studied. Has developed a heuristic to solve the problem of periodic rural postman without direction, but has not presented an exact model for the problem.

A mathematical model for linear programming suggested for a PCARP is presented by [8], for this are considered the minimum and maximum spacing between the executions of the task. The lower limits have been studied and a formal mathematical proof is presented. The proof shows that a lower limit of CARP also is an inferior limit for PCARP. A method of tabu search is developed and applied for the PCARP in adapted cases, from references of the CARP.

In [9] a simple classification of the PCARP is made and a problem in mixed arcs is proposed, suggesting a Memetic Algorithm (MA) to solve the problem. A Memetic Algorithm, according to the author is a hybrid genetic algorithm with a local search. The name was proposed by [10], being adopted in the work because the hybrid genetic algorithms are very diffuse, since the algorithm has been hybridized with many other techniques, such as neural networks and simulated annealing. A linear programming mathematical model is not stated, but computational tests have been executed to evaluate the proposed heuristic comparing with the periodical vehicle routing problem (PVRP).

In [11] has been proposed a model of linear programming for a periodical refuse collection problem. For this model has been determined the set of combinations of days on which the tasks must be accomplished. To evaluate the proposal, a closer heuristic insertion and a heuristic insertion of viability for inclusion has been developed, such as the insertion costs. A two-phased heuristic, on the first phase uses a good inferior limit to prepare a list of possible arcs, which is, a cluster, and, next, solves a routing problem for a single vehicle.

In [12] has been studied a road monitoring inspection problem in Quebec - Canada, in a context in which the planned route rarely is finished. A methodology has been developed to perform the task of data collection from the GPS tracking, combined with the routes planned within the geographical information system (GIS) to, later, use the mathematical algorithms to propose new routes. The study consists in a hierarchy of three classes of roads that have different standards of monitoring and a planning horizon of two weeks.

In [13] two algorithms have been applied to solve the PCARP. The first is a heuristic for better insertion and the second is called Scatter Search based on a local search. On this work has not been considered a mathematical linear programming model, but applied heuristic algorithms to solve the Periodic Capacitated Arc Routing Problem.

A combination of the ant's colony associated to a heuristic insertion to solve the PCARP is proposed by [14], getting robust results and with an acceptable performance. It is characterized as Routing Problem in Capacitated Arcs and Mixed Periodic because of the nature of the graph. Each service occurs according to a combination of days that satisfy the frequencies. An exact mathematical model has not been presented, however they say that the heuristic was able to find thirteen new better solutions for the periodic problem proposed.

In [15] has been developed a memetic algorithm to solve a PCARP. The authors proposed a mathematical model where the objective function is composed by a primary objective and a secondary objective. In this model, the primary objective is to minimize the number of vehicles in the time horizon and the secondary objective, the total cost. The primary objective is valiant by mnv and tc the total cost. So, the authors seeks for

$$min\, f(S) = \alpha.mnv(S) + tc(S)$$

where α is a sufficiently high number to guarantee a priority of number of vehicles. It has been verified that the primary objective mnv can hardly be improved with the existent research operators, such as [9] and [13]. To answer this question, a specific routes fusion procedure has been developed. This procedure is incorporated to the memetic algorithm structure. The proposed algorithm perform, first an improvement on the solution serving to the main objective to, later, make local searches.

An exact mathematical model of linear programming is given by [16] which not necessarily the condition defined by [9] always can be verified. The authors considered, for example, one week as planning period, where it was intended to plan the road's inspection for monitoring of ice and snow. Therefore, it may be wanted some arcs to be attended twice during the first five days and once during the weekend, being the days of service variable from one week to the other. This type of problem has been named as PCARP with irregular services (PCARP-I). It is proposed a two-phased algorithm to solve the problem. On the first phase, it is constructed the grouping of arcs without violating the route capacity and on the second phase, resolves the routing, considering only on vehicle.

In [17] has been presented a mathematical model for the routing problem in periodical capacitated arcs for monitoring of railway bars. Includes, in this model, a condition of penalty if the vehicle could not attend the arc or any delay happen and considered as capacity of the vehicle the fact of being able to attend only one stretch for the designated day. It has been compared the proposed models in literature and also a systemic evaluation between the model proposed by [16] and the one proposed by the author.

III. The Problem

In face of the accidents and incidents in dams that happened in the last years, there was a need for discussion and formulation of dam safety management models, involving the regulators, licensors, supervisory bodies, as well as the exploration companies and potentially affected communities. Accidents or incidents can be associated to the lack of commitment of the companies, with management procedures , with long term planning, of adequate engineering project, lack of specialized supervision, absence of operation manual, improvisation of the operation team, lack of inspections and periodical safety evaluation [18].

The inspection and monitoring of the dam's performance must go through a revaluation, to determine if the methods and the frequency of observation and monitoring are adequate and sufficient to detect any anomaly condition or instability in function of time. Still the revaluation must determine if the monitoring data have been regularly analyzed and used, to assure early detection of any potentially unsafe condition on the dam, relative to the water levels and the slopes of the reservoir [19].

According to [19], the inspections, monitoring of the barrage structure and the test of unloading facilities must be standardized. So, after the installation is recommended that each instrument be read at the same time of day: the instruments must be split in groups of observation in the same day and its reading must be scheduled with fixed frequency and itinerary. Must be provided standardization and guidelines for the establishment of the types of inspection to be performed; the purpose of each type of inspection and the frequency of them; the items to be inspected; the necessary documentation; the qualification and the training of the inspectors; and the procedures for the correction of deficiencies [19].

So, must ensure that the meter readers work as visual inspectors, covering several stretches and the dam's galleries, at least once a week. This recommendation is especially valid for the operational period. In this context, considering the security management plan of a dam highly instrumented, has been noticed the Periodic Capacitated Arc Routing Problem to form routes for monitoring the instruments installed along the dam, so allowing the standardization of data collection and fixed itinerary.

IV. Modeling Proposal

Being $G(V,E)$ a graph, where V is the set of vertices,$V = \{i \in \mathbb{N}; i = 1,2,\cdots N_V\}$, E the set of edges that compose the graph $E = \{e \in \mathbb{N};\ e =$

$1,2,\cdots,N_E\}$, TA the set of equipments $TA=\{k\in N;\ k=1,2,\cdots,N_{TA}\}$, Q_k the quantity of equipments of the type k with which must be collected readings in all the edges of the graph.

For each edge (i,j) is associated a distance d_{ij}. There is a finite quantity of instruments installed of the type k given by q_{kij} the $q-instrumento$ of type k installed on the edge (i,j). Each type of instrument has a frequency of reading, and will be denoted by $f_{\mathrm{k,i,j}}$ the frequency of instrument reading k on the edge (i,j). The frequency for each type of instrument will be the same for all the edges.

In the problem addressed as motivation of his work, for each type of instrument must be used a type of reading equipment, being the available equipments number varied. Being A_{p_k} the quantity of available equipments for reading of the instrument that uses the equipment type k. The vehicle of transportation considered in the problem of this work is the meter reader. So, each equipment k that the meter reader v can operate it is set the value 1, otherwise, for equipments that the meter reader is not enabled to carry due to the weight (the weight is the restriction of the vehicle's capacity adapted to the problem addressed in this work), is the sum of weights of equipments which are enabled to carry plus one. Denote by A_{H_v} the set of equipments that the meter reader v is enabled to carry.

$$P_{k,v}=\begin{cases}1 & se\ k\in A_{H_V}\\ 1+\sum_{s\in A_{H_v}} P_{s,v} & se\ k\notin A_{H_v}\end{cases}$$

The carrying capacity for the meter reader v is given by C_{LV}, being their capacity the maximum number of equipments that the meter reader v can carry within the equipments he's able to operate.

For each task it is considered a mean time for its execution and average speed for each meter reader, in other words, must be considered γ_k the average time for the reading of the instrument $k$and V_{mv} the average speed in meters by second of the meter reader v. Each meter reader have working hours W_v, representing the capacity in time to develop the tasks.

The planning horizon is given in a total of D days, so $d=1,2,\cdots,D$. The set of possible collection days is denominated by the set of allowed combinations. This combinations represent a set of days that satisfy the interval between readings, instrument's frequency, of a same equipment at a given arc. Denote by $\Psi_k=\{\psi_1,\psi_2,\cdots,\psi_n\}$ the n possible combinations of days for which are possible the collection of data for the instrument k.

The indices, parameters and variables that will be used in the mathematical model are presented in the Table1.

Table 1: Description of the indices, parameters and variables of the mathematical model

Symbology		**Description**
Indices	i,j,r,s	knots;
	d	day of the period;
	k	instrument;
	v	meter reader;
	ψ	allowed combination;
Parameters	$a_{d,\psi,k}$	If the day d belongs to the combination ψ in relation to the instrumenttype k, assume value 1. Otherwise zero;
	$d_{i,j}$	Distance between knots i and j;
	A_{p_k}	Quantity of reading equipments of the instrument type k;
	CL_v	Carrying capacity for the meter reader v;
	$P_{k,v}$	Associated weight to the reading equipment type k in relation to the meter reader v;
	N_{TA}	Number of total distinct instruments;
	γ_k	Time to read the tool type k;
	$q_{k,i,j}$	Quantity of instruments installed on the edge (i,j);
	$V_{m,v}$	Average speed of the meter reader v;
	$\|E\|$	Number of arcs that compose the graph;
	W_v	Capacity of working hours for the meter reader v;
Variables	$x_{i,j,v,d}$	Counts the number of times the edge (i,j) it is traveled by the meter reader v on day d;
	$l_{i.j.v,k,d}$	If the edge (i,j) is attended by the meter reader v on day d for data collection of the instrument type k assumes value 1. Otherwise zero;
	$z_{v,d,k}$	Counts the number of equipments used on day d by the meter reader v;
	$w_{i,j,d,k}$	If the edge (i,j) is visted on day d in relation to the instrument k takes value 1. Otherwise zero;
	$m_{i,j,\psi,k}$	If the edge (i,j) is chosen at the combination ψ in relation to the instrument k takes

		value 1. Otherwise zero;
	$t_{v,d}$	If the meter reader v attended a task on day d assumes value 1. Otherwise zero.

It is defined the three dimensional matrix $a_{d,\psi,k}$ in the following way, 1If the day d belongs to the combination ψ in relation to the instrument k, 0 otherwise.

Given these considerations, the formulation is given below. Minimize z given by.

$$z = mim \sum_{i=1}^{N_V} \sum_{j=1}^{N_V} \sum_{d=1}^{D} \sum_{v=1}^{NV} \mathbf{d_{i,j}} . \mathbf{x_{i,j,v,d}} \quad (1)$$

Subject to the following restrictions.

$$\sum_{\psi \in \Psi_K} \left(m_{i,j,\psi,k} + m_{j,i,\psi,k} \right) = 1 \qquad \begin{cases} (i,j) \in E \\ k = 1, 2, \cdots, N_{TA} \end{cases} \quad (2)$$

$$w_{i,j,d,k} = \sum_{\psi \in \Psi_K} m_{i,j,\psi,k} . a_{d,\psi,k} \qquad \begin{cases} d = 1, 2, \cdots, D \\ (i,j) \in E \\ k = 1, 2, \cdots, N_{TA} \end{cases} \quad (3)$$

$$\sum_{v=1}^{NV} z_{v,d,k} \leq A_{p_k} \qquad \begin{cases} k = 1, 2, \cdots, N_{TA} \\ d = 1, 2, \cdots, D \end{cases} \quad (4)$$

$$\sum_{k=1}^{N_{TA}} P_{k,v} . z_{v,d,k} \leq CL_v \qquad \begin{cases} v = 1, 2, \cdots, NV \\ d = 1, 2, \cdots, D \end{cases} \quad (5)$$

$$\sum_{v=1}^{NV} l_{i,j,v,d,k} \leq w_{i,j,d,k} \qquad \begin{cases} e_{i,j} \in E \;\; ou \; a_{i,j} \in A \\ k = 1, 2, \cdots, N_{TA} \\ d = 1, 2, \dots, D \end{cases} \quad (6)$$

$$N_{TA} . x_{i,j,v,d} = \sum_{k=1}^{N_{TA}} l_{i,j,v,d,k} \qquad \begin{cases} e_{i,j} \in E \;\; ou \; a_{i,j} \in A \\ v = 1, 2, \cdots, NV \\ d = 1, 2, \dots, D \end{cases} \quad (7)$$

$$\sum_{i=2}^{N_V} x_{1,i,v,d} \leq 1 \qquad \begin{cases} d = 1, 2, \dots, D \\ v = 1, 2, \dots, NV \end{cases} \quad (8)$$

$$\sum_{j=1}^{N_V} x_{j,i,v,d} = \sum_{j=1}^{N_V} x_{i,j,v,d} \qquad \begin{cases} i = 2, 3, \dots, N_V \\ v = 1, 2, \cdots, NV \\ d = 1, 2, \dots, D \end{cases} \quad (9)$$

$$|E| z_{v,d,k} \geq \sum_{i=1}^{N_V} \sum_{j=1}^{N_V} l_{i,j,v,d,k} \qquad \begin{cases} k = 1, 2, \dots, N_{TA} \\ v = 1, 2, \cdots, NV \\ d = 1, 2, \dots, D \end{cases} \quad (10)$$

$$N_{TA} . t_{v,d} \geq \sum_{k=1}^{N_{TA}} z_{v,d,k} \qquad \begin{cases} d = 1, 2, \dots, D \\ v = 1, 2, \dots, NV \end{cases} \quad (11)$$

$$\sum_{v=1}^{NV} \sum_{i=2}^{N_V} x_{i,1,v,d} = \sum_{v=1}^{NV} t_{v,d} \qquad \{ d = 1, 2, \dots, D \quad (12)$$

$$\sum_{k=1}^{N_{TA}} \sum_{(i,j) \in E} \gamma_k . q_{k,i,j} . l_{i,j,v,d,k} + \sum_{(i,j) \in E} \frac{d_{i,j}}{V_{m,v}} . x_{i,j,v,d} \leq W_v \qquad \begin{cases} d = 1, 2, \dots, D \\ v = 1, 2, \dots, NV \end{cases} \quad (13)$$

$$\sum_{r \in Q} \sum_{s \notin Q} x_{r,s,v,d} \geq \frac{1}{|Q|^2 - |Q|} \sum_{i,j \in Q} x_{i,j,v,d} \qquad \begin{cases} v = 1, 2, \cdots, NV \\ d = 1, 2, \dots, D \\ \forall \, Q \subset \{ 2, 3, \dots, n \} \\ |Q| \geq 2 \end{cases} \quad (14)$$

$$x_{i,j,v,d} \in \mathbb{Z}_+ l_{i,j,v,d,k};\; t_{v,d};\; z_{v,d,k} \in \{0,1\} \qquad m_{i,j,\psi,k};\; w_{i,j,d,k} \in \{0,1\} \quad (15)$$

The objective function given in (1) seeks to minimize the total distance traveled by the meter reader along the planning horizon.

The restrictions (2) guarantees that only one combination will be designated for the reading of the instrument k installed at arc (i, j) and in only one direction.

The restriction (3) assures that, for each arc (i, j), the readings occur on the day wich has been designated, based on the chosen combination.

The restriction (4) guarantees that the quantity of equipments type k used by all the meter readers in one day d of reading be lower than the allowed quantities.

The restriction (5) guarantees that the capacity of the meter reader guarantees that the capacity of the meter reader is not extrapolated in relation to the equipments he carries to collect data in one day of reading.

The restriction (6) guarantees that the meter reader will travel the arc (i, j) for data collection of the instrument type k, if the arc is allocated for that day, satisfying the condition of combination allowed for that day.

The restriction (7) guarantees that the arc (i, j) will be traveled by the meter reader v on day d if he has any equipment and the arc have been designated for that day.

The restriction (8) guarantees that the meter reader be used once in the day and, if he have other equipments, that he will start the reading from the same arc.

The restriction (9) guaranteesthe flow's continuity.

The restriction (10) assures that the equipment type k will be carried by the meter reader v at the day d.

The restriction (11) counts the number of meter readers being used on the day d.

The restriction (12) guarantees that all the meter readers return to the office.

The restriction (13) guarantees that the time to accomplish the readings will not go longer than the working hours for each meter reader.

The restriction (14) prevents the formation of isolated depositing subcycle.

The restriction (15) guaranteesthat the variables are integer and assures that the variables are binary.

V. Instances of Generation

The reference values available for this problem constitute of four random sets of problems called $GRP1$,$GRP2$, $GRP3$ and $GRP4$. In the www.engprod.ufpr.br/GTAO/InstanciasPCARP/InstanciasPCARP.rar all the instances can be found.

The instances have been generated randomly using procedures similar to [16]. To define the number of arcs, considering three different sets: ($\{5 - 20\}$, $\{20 - 80\}$ and $\{80 - 200\}$). To the first set, two random numbers have been created, defining the number of arcs of two different graphs; for the second group three random numbers have been created to define the number of arcs of three distinct graphs; and for the third group it was created two graphs similarly to the above mentioned. Denotes by n_a the number of arcs obtained by a random integer number within each intervals. For each case the number of two knots have been obtained within the numbers

$$\left\{\lceil r_n \rceil \,;\, \frac{n_a}{2}\right\}$$

wherer_n is the positive root of the equation

$$n^2 - n - n_a = 0, \ \ so, \qquad r_n = \frac{1 + \sqrt{1 + 4.n_a}}{2}.$$

The formation of the arcs between two knots i and j is made up as follows: For all $i < j$ It was randomly assigned number between 0 and 1. If the number assigned was lower than 0.51 the edge (i, j) did not exist, otherwise the edge was determined. This procedure would repeat until the number of arcs were determined.

For all the instances, the time horizon have been defined in 5 days (Monday to Friday). The days when necessarily must occur instrument readings respecting the frequency of each reading was defined as empirically pre-determined allowed combination. The time horizon starts on Monday, denoted 1, and so on until Friday, denoted by 5.

Table 2: Set Permitted Combinations

Instruments	Allowed Combinations	Total
1	(1, 3), (2, 4)	4
2	(1, 4), (2, 5)	
1	(1), (2), (3), (4), (5)	9
2	(1, 3), (2, 4)	
3	(1, 4), (2, 5)	
1	(1), (2), (3), (4), (5)	10
2	(1, 3), (2, 4)	
3	(1, 4), (2,5)	
4	(1, 5)	
1	(1), (2), (3), (4), (5)	12
2	(1, 3), (2, 4), (3, 5)	
3	(1, 4), (2,5)	
4	(1, 5)	
5	(1, 3, 5)	
1	(1), (2), (3), (4), (5)	16
2	(1, 3), (2, 4), (3, 5)	

3	(1, 4), (2,5)
4	(1, 5)
5	(1, 3, 5)
6	(1, 2), (2, 3), (3, 4), (4, 5)

For problems with 2 tasks by arc, four combinations were allowed to carry out the assigned tasks, two combinations allowed for each type of task. For problems with 3 tasks were assigned nine allowed combinations and for the other cases of problems with larger dimensions the information is presented in Table 2.

The maximum number of different tasks defined by 6, the minimum number of tasks has been defined as 2, so as to satisfy the condition of multiple tasks. For all graph tasks were assigned between the minimum and the maximum value, randomly generated following a discrete uniform distribution between 2 and 6.

The number of instruments for each task was generated using the same distribution of the number of tasks cited. Since, for the first, second and sixth graph, ordered by creation presented above, the number of instruments for each arc is generated between$\{2 - 10\}$, for the third between $\{5 - 15\}$, for the fourth and fifth graph between $\{2 - 7\}$ and for the seventh $\{2 - 13\}$.

The distance route was randomly generated considering the number of arcs. If $n_a \in \{5 - 29\}$ for each edge was generated a random number α in the interval $[0.70; 1]$ and the distance defined by $d = \lceil \alpha . n_a \rceil + 2$. If $n_a \in \{30 - 39\}$ for each edge it was generated a random number αin the interval $[0.40; 1]$ and the distance defined by $d = \lceil \alpha . n_a \rceil + 4$. If $n_a \in \{40 - 50\}$ for each edge was generated a random number αin the interval [0.40; 1] define the distance $= \left\lceil \frac{\alpha . n_a}{2} \right\rceil$. If $n_a \in \{51 - 80\}$ for each edge was generated a random number α in the interval $[0.15; 0.8]$ and the distance defined by $d = \left\lceil \frac{\alpha . n_a}{2} \right\rceil$. If $n_a \in \{81 - 120\}$ for each edge was generated a random number αin the interval $[0.10; 0.40]$ and the distance defined by $d = \left\lceil \frac{\alpha . n_a}{2} \right\rceil$. If $n_a \in \{121 - 220\}$ for each edge was generated a random number α in the interval $[0.05; 0.25]$ and the distance defined by $d = \left\lceil \frac{\alpha . n_a}{2} \right\rceil$. The conditions to define the distances seeks to guarantee the feasibility regarding the ability of the vehicle in relation to the available working hours that does not exceed 7 hours. All random generations regarding the distance follow an uniform continuous distribution between the limits mentioned.

The average speed of walking was defined in $2.5 m/s$. Studies show that the average can be lower, however it was considered the average value between the data values of the upper and lower average described in [20], which states that the normal walking of a person is moving with a speed between $5.63 km/h (1.56\ m/s)$ and $201.0\ m/min (3.35\ m/s)$.

The average time to collect the data for each distinct type of instruments have been set by one of the following values: 35, 45, 30, 50, 40, 30, in seconds. The values have been defined through observations of the problem.Values were defined by motivating problem observations and will be used randomly on every reading.

It should be emphasized that the solution to the difficulty of PCARP should be more strongly, the total number of services, than the number of arcs of the graph. According to [5], problems where the number of services exceeds the amount 200 per time horizon are considered very large.

VI. Computational Results

The tests were performed on four types of different computers. For problems of the set $GRP1$ and $GRP2$ two softwares have been used LINGO 12 and ILOG CPLEX and comparisons with the use of three of four different computers.

The Table 3-7 have: The columns in the order presented, are: file name, number of knots (Nº Nós), number of arcs (Nº Ar), number of tasks (Nº Ins.), number of vehicles (Nº L.), number of allowed combinations (Nº C.), vehicle's capacity in relation to time (Te. Tra.), Maximum number of tasks that can be executed simultaneously by vehicle (Cap. Car.), inferior limit presented by the software LINGO 12 (L. Inf.), value of objective function presented by software LINGO 12 (F. O.), processing time in second by software LINGO 12 using a computer (T. -S.), GAP presented by software CPLEX (GAP (\%)), value of objective function presented by software ILOG CPLEX (F. O.), processing time in seconds by software ILOG CPLEX using a computer (T.-S).Tests were performed on computers Intel(R) Core(TM) $i3-2310M$ CPU 2.10 GHz, ram memory 4.00 GB and operational system Windows 64 bits, and Intel(R) Core(TM) $i7-4500U$ CPU 1.80 GHz, ram memory 8.00 GB and operational system Windows 64 bits.

The software LINGO presented a lower performance than ILOG CPLEX. Of 17 problems presented 3 results with LINGO were superior to CPLEX for the objective function, the problems $Prb17$, $Prb14$ and $Prb9$. For the problem $Prb17$ the software had to be stopped before reaching the great value.These results are presented in table 3.

All the results presented with * represent the values obtained, although do not represent the great value during the processing time. For all the problems where the great value was reached is noted that the convergence to the software ILOG CPLEX was faster.

Table 3: Computational results for the first group of problems

Name	Nº Nós	Nº Ar.	Nº Ins.	Nº L.	Nº C.	Te. Tra.	Cap. Car.	LINGO 12 (i7)			ILOG CPLEX (i3)		
								L. Inf.	F. O.	T. –S.	GAP (%)	F. O.	T.-S.
Prb1	6	16	2	1	4	3600	2	351	351	4.00	0.00	351	0.01
Prb2	6	16	3	1	9	3600	2	351	351	31.00	2.42	351	1.59
Prb3	6	16	3	1	9	3600	3	351	351	60.00	2.56	351	1.20
Prb4	6	16	4	1	10	3600	2	468	468	20.00	0.64	468	0.53
Prb5	6	16	5	1	12	3600	3	585	585	18.00	4.13	585	0.83
Prb6	6	16	5	2	12	3600	2	505	585*	3704.00	3.10	585	72.52
Prb7	6	16	6	1	16	3600	3	585	585	38.00	4.75	585	0.65
Prb8	6	16	6	2	16	3600	2	553	702*	33431.00	0.60	702	3169.75
Prb9	6	16	6	2	16	3600	3	511	585*	13327.00	0.35	578	7149.31
Prb10	6	18	2	1	4	3600	2	399	399	9.00	6.02	399	0.39
Prb11	6	18	3	1	9	3600	2	399	399	130.00	6.52	399	1.73
Prb12	6	18	4	1	10	3600	2	532	532	99.00	7.63	532	0.89
Prb13	6	18	5	1	12	7200	3	532	532	39.00	8.30	532	0.58
Prb14	6	18	5	2	12	3600	2	530	687*	14256.00	0.45	665	7251.19
Prb15	6	18	6	1	16	7200	3	531	532	42.00	4.51	532	1.58
Prb16	6	18	6	2	16	3600	2	590	798*	220577.00	0.24	798	192825.19
Prb17	6	18	6	2	16	3600	3	607	693*	68881.00	0.28	676	106281.44

Table 4: Computational results for the second group of problems

Name	Nº Nós	Nº Ar	Nº Ins.	Nº L.	Nº C.	Te. Tra.	Cap. Car.	LINGO 12 (i5)			ILOG CPLEX (i3)		
								L. Inf.	F. O.	T. –S.	GAP (%)	F. O.	T.-S.
Prb18	9	30	2	1	4	7200	2	1101	1101	1942.00	2.54	1101	11.30
Prb19	9	30	2	1	4	10800	2	1101	1101	2470.00	1.58	1101	33.41
Prb20	9	30	3	1	9	7200	2	1101	1101	9545.00	0.56	1101	96.28
Prb21	9	30	3	1	9	10800	3	1101	1101	8405.00	0.98	1101	52.29
Prb22	9	30	4	1	10	10800	2	1508	1508	7353.00	0.65	1508	53.56
Prb23	9	30	5	1	12	14400	3	1850	1850	1418.00	0.42	1850	92.45
Prb24	9	30	5	2	12	10800	2	1333	2318*	92110.00	19.00	1925*	4723.72
Prb25	9	30	6	1	16	14400	3	1885	1885	6514.00	1.10	1885	30.80
Prb26	9	30	6	2	16	10800	2	1545	3144*	14458.00	30.72	2308*	10381.93
Prb27	9	30	6	2	16	10800	3	1276	1755*	10730.00	9.39	1668*	12643.66
Name	Nº Nós	Nº Ar	Nº Ins.	Nº L.	Nº C.	Te. Tra.	Cap. Car..	LINGO 12 (i5)			ILOG CPLEX (i7)		
								L. Inf.	F. O.	T. –S.	GAP (%)	F. O.	T.-S.
Prb28	11	40	2	1	4	10800	2	705	705	4364.00	0.52	705	19.52
Prb29	11	40	3	1	9	10800	2	705	705	13058.00	0.61	705	33.11
Prb30	11	40	3	1	9	10800	3	705	705	22298.00	0.30	705	52.97
Prb31	11	40	4	2	10	25200	2	801	1193*	39653.00	1.92	964*	5177.48
Prb32	11	40	4	2	10	25200	3	801	1153*	39563.00	1.98	964*	4522.56
Prb33	11	40	5	2	12	25200	2	***	***	91090.00	***	infeasible	1181.42

The Table 4 was constructed analogously to Table 3. For the group of $GRP2$ problems computational tests were performed on computers with the following settings, Intel(R) Core(TM) $i5-650$ CPU 3.20 GHz, ram memory 4.00 GB and operational system Windows 64 bits, Intel(R) Core(TM) $i3-2310M$ CPU 2.10 GHz, ram memory 4.00 GB and operational system Windows 64 bits, and Intel(R) Core(TM) $i7-4500U$ CPU 1.80 GHz, ram memory 8.00 GB and operational system Windows 64 bits.

The problem $Prb33$ is an unworkable problem, such impossibility is related to the number of meter readers to perform tasks. In the presented problems, although the number of arcs are few, it is observed that given the number of tasks to be performed becomes a problem with high computational complexity.

Table 5: Computational results for the third group of problems

Name	Nº Nós	Nº Ar	Nº Ins.	Nº L.	Nº C.	Te. Tra.	Cap. Car.	CPLEX			PC
								GAP (%)	F. O.	T.-S.	
Prb34	11	40	5	3	12	25200	3	12.10	1036*	98254.22	(i7)
Prb35	11	40	5	4	12	25200	2	35.01	1393*	395644.00	(i7)
Prb36	12	52	2	3	4	25200	2	4.00	889*	54347.80	(i3)
Prb37	12	52	2	3	4	25200	3	7.19	902*	22983.77	(i3)
Prb38	12	52	3	4	9	25200	2	9.68	885*	139078.60	(i3)

For the group of problems $GRP3$ tests were performed in computers $i3$ and $i7$ with the settings described above. The computational results for the group GRP3 are shown in Table5. For this group tests were performed using only the software ILOG CPLEX, the problems with 40 arcs the processor used was $i7$ and for the problems with 52 arcs the processor used was $i3$ and Table 6 were performed with computers Intel(R) Xeon(R) CPU $E5-2650$,2.00GHz ram memory 32 GB, operational system Windows 64 bits.

Table 6: Computational results for the fourth group of problems

Name	Nº Nós	Nº Ar	Nº Ins.	Nº L.	Nº C.	Te. Tra.	Cap. Car.	CPLEX		
								GAP (%)	F. O.	T.-S.
Prb39	11	40	5	4	12	25200	3	13.90	1036*	3293.09
Prb40	11	40	6	3	16	25200	3	10.94	1036*	4601.70
Prb41	11	40	6	4	16	25200	2	43.76	1631*	148361.16
Prb42	11	40	6	4	16	25200	3	13.39	1036*	63333.13
Prb43	12	52	3	4	9	25200	3	19.41	974*	16945.09
Prb44	12	52	4	4	10	25200	3	13.54	1315*	1325.74
Prb45	12	52	4	4	10	25200	2	11.43	1262*	7323.78
Prb46	12	52	5	4	12	25200	2	31.22	1659*	66428.36
Prb47	12	52	5	4	12	25200	3	13.41	1312*	399.05
Prb48	12	52	6	4	16	25200	3	15.60	1346*	65118.50
Prb49	12	52	6	4	16	25200	2	49.89	2311*	73516.03
Prb50	12	52	6	5	16	25200	2	50.15	2324*	91351.00

The computational results here show the computational complexity compared to the processing time even for problems with a few arcs.

To the problems presented so far you can see that the computational complexity has characteristics related to number of meter readers and the number of concurrent tasks for them to be carried out. Note the problems $Prb41$and $Prb42$. The processing time for the problem $Prb41$ is higher than twice the processing time for the problem $Prb42$, although the GAP for the problem $Prb41$ is higher than three times the GAP for the problem $Prb42$. This same observation can be made about the problems $Prb46$and $Prb47$. The more tasks simultaneously the meter reader is able to run implies a lower computational complexity. The computational complexity is also related to the number of meter readers, note the processing time to problems $Prb49$and $Prb50$ for wich is used 4 and 5 meter readers respectively.

For the group of problems $GRP4$ with numbers lower than 100 arcs tests were performed with computers Intel(R) Xeon(R) CPU $E5-2650$,2.00GHz ram memory 32 GB, operational system Windows 64 bits. The computational results for the group $GRP4$ are shown in the Table 7. For this group tests were performed using only the software ILOG CPLEX.

Table 7: Computational results for the fourth group of problems to over **100** arches

Name	Nº Nós	Nº Arc	Nº Ins.	Nº L.	Nº C.	Te. Trab.	Cap. Car.	CPLEX		
								GAP (%)	F. O.	T.-S.
Prb51	16	106	2	4	4	25200	2	23.53	2252*	8604.05
Prb52	16	106	2	3	4	25200	2	25.79	2321*	2066.19
Prb53	16	106	3	4	9	25200	2	14.66	2197*	254824.22
Prb54	16	106	3	4	9	25200	2	37.93	2793*	2303.11
Prb55	16	106	3	4	9	25200	3	16.17	2337*	75428.68
Prb56	16	106	4	4	10	25200	2	13.82	3162*	72922.16
Prb57	16	106	5	5	12	25200	2	****	****	254351.64
Prb58	16	106	5	4	12	25200	3	9.91	3059*	66022.49
Prb59	16	106	6	4	16	25200	2	****	****	256352.23
Prb60	16	106	6	4	16	25200	3	13.31	3179*	72308.51
Prb61	16	106	6	5	16	25200	2	51.16	5716*	194824.83
Prb62	20	180	2	4	4	25200	2	10.63	3941*	255280.55
Prb63	20	180	2	5	4	25200	2	17.41	4109*	170876.27
Prb64	20	180	3	4	9	25200	2	23.19	4404*	144607.97
Prb65	20	180	3	4	9	25200	3	19.58	4555*	237322.37
Prb66	20	180	4	4	10	25200	3	15.28	5841*	236609.52
Prb67	20	180	4	4	10	25200	2	11.51	5601*	254403.15
Prb68	20	180	4	5	10	25200	2	28.67	6864*	262669.01
Prb69	20	180	5	5	12	25200	2	36.19	7932*	181415.89
Prb70	20	180	5	4	12	25200	3	11.24	5687*	262309.41
Prb71	20	180	6	5	16	22800	3	29.40	7150*	263715.74

VII. Conclusions

In this article, is presented a proposal definition of a new problem: the Multiple tasks Periodic Capacitated Arc Routing Problem in the time horizon (MTPCARP). Some real applications can be found, for example, dam safety monitoring.

All studies in the literature that studies periodic routing problems in knots as in arcs within a time horizon do not address the case of multiple tasks with the possibility of simultaneous execution. The only work that addresses routing problems with multiple tasks is given by [21] who has studied the problem of collection of recyclable waste, which involved three different types of waste and must be collected separately, the study proposes a heuristic for determining the solutions.

The problems evaluated here were randomly generated, since there are no literature cases of problems with multiple tasks. The results presented, seek to propose a set of problems that can be compared with new motions given by future literature.

The main objective was to evaluate the proposed modeling, as the computational results shows that the proposed modeling is coherent according to the seeking to minimize the total traveled distance along the time horizon assigning the tasks to be executed, satisfying their periodicities.

As is proposed for future applications, investigate other forms of determination for best solutions through development of heuristics and/or metaheuristics to solve the problem.

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